

Critical Statistical Charge for Anyonic Superconductivity

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Abstract

We examine a criterion for the anyonic superconductivity at zero temperature in Abelian matter-coupled Chern-Simons gauge field theories in three dimensions. By solving the Dyson-Schwinger equations, we obtain a critical value of the statistical charge for the superconducting phase in a massless fermion-Chern-Simons model.

One of the remarkable features of 2+1 dimensional gauge theories is that they admit a parity (P) and time-reversal (T) violating Chern-Simon (CS) term of the gauge field [1]. Via a CS coupling, the charged planar particles are attached with magnetic fluxes and, depending on the strength of the effective CS coupling, the spin and statistics of the particles are transmuted. Therefore, coupling a CS field to an ordinary fermion or boson field provides a description of anyons [2]-[7]. The concept of anyons has found its application first in the fractional quantum Hall effects [8]-[10]. It was also conjectured [11] that the gas of charged anyons would exhibit the property of superconductivity. In the last few years, much progress has been made along the line [12]-[21]. In particular, a criterion for the anyonic superconductivity has been established. In the language of the CS matter field theory, this criterion is expressed as [17]: superconductivity (at zero temperature) occurs if and only if the renormalized CS term vanishes. This statement is based on the observation that, when the renormalized CS term is absent, there exists a Goldstone pole in the effective low-momentum lagrangian, and the remaining path integral may be rewritten in the conventional Landau-Ginzburg form (in the London limit). Moreover if one chooses the bare CS coefficient to be unit, leaving the CS matter coupling constant e (the statistical charge) free, and denotes the vacuum polarization associated with the CS term by $\Pi_o = \Pi_o(e, p^2 = 0)$, the criterion takes the form

$$\tilde{\Pi}_o = 1 + \Pi_o = 0 \tag{1}$$

Namely, the matter-induced CS term cancels out the bare one exactly. Eq.(1) determines the critical statistical charge, e_c , of the superconducting phase of the system.

In this paper we examine the criterion in certain field theory models. In particular, we will discuss a method of using the Dyson-Schwinger (DS) equations to calculate the critical statistical charge, e_c , for massless theories. To be concrete,

we will use the CS massless fermion theory as an example. A parallel discussion for the CS massless boson theory is straightforward.

We start with a brief review of relevant perturbative results and point out the difference between the cases of the massive and massless theories. Let us consider the $U(1)$ gauge theory with a CS field minimally coupled to a fermion field and a boson field in the three dimensional Euclidean space ($g_{\mu\nu} = \delta_{\mu\nu}$):

$$\mathcal{L} = \psi^\dagger [\gamma_\mu (\partial_\mu - ie a_\mu) + iM] \psi + (\partial_\mu + ie a_\mu) \phi^* (\partial_\mu - ie a_\mu) \phi + m^2 \phi^* \phi - i\epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \quad (2)$$

where the two-by-two gamma matrices γ_μ are anti-hermitean, $\gamma_\mu^\dagger = -\gamma_\mu$; ψ is a two-component spinor, ϕ is a complex scalar, and a_μ is the Chern-Simons gauge field. M and m are the fermion and boson mass, respectively. The coupling constant e is dimensionless by naive powercounting (For simplicity we use e to denote the couplings for both the CS scalar and CS spinor, knowing the two interactions are not necessarily to have the same strength). The symmetries of the theory have been discussed in [1]. In particular, the charge conjugation (C) transformation leaves the lagrangian invariant, while the fermion mass and the Chern-Simons terms are both variant under parity (P) or time reversal (T). But CPT symmetry holds.

For the massive matter fields (M and m are non-zero), there is a no-renormalization theorem [22][23]: there is no radiative correction to the CS term beyond one-loop. Consequently, one needs to compute only one-loop diagrams to determine Π_o . The one-loop results are [4]

$$\Pi_o = \Pi_o^{l=1} \equiv 0, \quad \text{from a massive boson} \quad (3)$$

$$\Pi_o = \Pi_o^{l=1}(p^2 = 0) = -\frac{e^2}{4\pi} \text{sign}(M), \quad \text{from a massive fermion} \quad (4)$$

From the above results, one may draw the conclusion that the massive fermions in the minimal coupling induce a CS term of the proper sign and inevitably exhibit superconductivity at some critical value of the statistical charge, while the massive bosons do not.

On the other hand, if the coupled matters are massless ($M = m = 0$), the no-renormalization theorem is invalid [23]. Explicit computation shows that massless matters, either massless bosons or massless fermions, do not contribute to the CS term at one loop, but make finitely contributions to it at two-loop [24]:

$$\Pi_o^{l=2} = -\frac{e^4}{16}\left(\frac{1}{4} - \frac{1}{\pi^2}\right) \quad \text{from a massless boson} \quad (5)$$

$$\Pi_o^{l=2} = -\frac{e^4}{4}\left(\frac{1}{16} + \frac{1}{\pi^2}\right) \quad \text{from a massless fermion} \quad (6)$$

Notice that, in the massless cases, Π_o is independent of the external momentum p . Four loop and higher order corrections are expected. We see now that the results from the massless matters are even qualitatively different from those of the massive ones [25]. In particular, a CS massless boson theory obviously exhibits the anyonic superconductivity too.

Interesting enough, the CS massless matter field theories at the critical points may be regarded as models of the P and T conserved superconductivity. Indeed, in the superconducting phase, the renormalized CS term - the only term that violates P and T - vanishes and the P and T symmetries are recovered. Saying so, we have actually assumed that the massless matter fields, especially fermion fields, in the CS quantum theory remain massless, a point we will further address later.

From the two-loop results, Eqs.(5) and (6), it is attempting to calculate the critical values of the statistical charge. These are $e_c = \pm 2.223$ for a fermion and $e_c = \pm 3.221$ for a boson. The two-loop results of such strong critical couplings raise concern on whether a perturbation expansion in e is applicable for studying the physics near and in the superconducting phase. Therefore, in this case, non-perturbative methods must be used. The one we will use is to solve the DS equations.

The DS equations, in principle, contain all information about the quantum field theory studied, and provide a natural non-perturbative scheme to study its

dynamics. However, since the DS equations are a set of infinitely many coupled equations, one needs to make certain truncation to get a tractable subset. Some features of (2+1) dimensional QED without or with a CS term have been studied by using appropriate approximations in the DS equation for the fermion propagator [26]-[30]. To the problem we are considering, it is necessary (and sufficient) to consider the DS equations for the two-point Green functions $\Delta_{\mu\nu}(p)$ for the CS field and $iS(p)$ for the fermion field (from now on, we focus on the CS massless fermion theory). Denoting the inverse of the two two-point Green functions by $\Pi_{\mu\nu}(p)$ and $iS^{-1}(p)$, respectively, we have the two DS Equations

$$\Pi_{\mu\nu}(p) = \Pi_{\mu\nu}^0(p) - e^2 \int \frac{d^3k}{(2\pi)^3} \gamma_\mu S(k+p) \Gamma_\nu(k+p, k) S(k) \quad (7)$$

$$S^{-1}(p) = S_0^{-1}(p) - e^2 \int \frac{d^3k}{(2\pi)^3} \gamma_\mu S(k+p) \Gamma_\nu(k+p, p) \Delta_{\nu\mu}(k) \quad (8)$$

where $\Gamma_\mu(p, q)$ is the full fermion-photon-fermion three-point function (the vertex function), which we will discuss in details later; and the bare quantities (in the Landau gauge) are

$$\Pi_{\mu\nu}^0(p) = \epsilon_{\mu\nu\sigma} p_\sigma, \quad iS_0^{-1}(p) = ip \cdot \gamma \quad (9)$$

$$\Delta_{\mu\nu}^0(p) = -\frac{\epsilon_{\mu\nu\sigma} p_\sigma}{p^2}, \quad iS_0(p) = i\frac{p \cdot \gamma}{p^2} \quad (10)$$

At this point, we would like to make several remarks. First of all, with only a dimensionless coupling constant e , the CS fermion theory is renormalizable. We will use the regularization by dimensional reduction, in which one performs a dimensional continuation of the integration measure in all Feynman integrals, but the vector and the tensor quantities including the Levi-Civita tensor $\epsilon_{\mu\nu\sigma}$ are always treated as if they are strictly three dimensional [24][31]. Secondly, we have chosen the Landau gauge for the gauge fixing. It is known that in this gauge choice the CS matter theory is explicitly free of infrared divergence [32]. Indeed, in the Landau gauge, the CS propagator takes the form of p^{-1} as $p \rightarrow 0$, as shown

in Eq.(10), contrast to p^{-2} for the propagator of the gauge field in the conventional QED₃. Third, it has been found that the β -function for the CS coupling vanishes up to three loops in various kinds of CS matter theories [24][31]. Since the kinetic CS action is of topological nature, it is reasonable to conjecture that the β -function vanishes to all orders. For a massless CS theory, whose lagrangian is scale and (global) conformal invariant, the vanishing β -function implies that the scale of conformal symmetries survive quantization and renormalization [31]. Consequently, all physical quantities are independent of the renormalization scale, μ (with dimension of mass), which is routinely introduced with the regularization scheme. In particular, the matter field and the CS field receive no radiative mass corrections. To see this, let $m(e, \mu)$ be the renormalized mass of, say, the matter field. By dimensional argument, the renormalized mass should be μ times a function of e [33]. Since any physical quantity is invariant under renormalization group transformation, we have

$$0 = \mu \frac{d}{d\mu} m(e, \mu) = m + \beta \frac{\partial}{\partial e} m \quad (11)$$

It shows that $m = 0$ when $\beta = 0$. Finally, we list the one loop corrections to the polarization tensor of the “photon”, the fermion self-energy and the vertex function:

$$\Pi_{\mu\nu}^{l=1}(p) = -\frac{e^2}{16p} P_{\mu\nu}, \quad i\Sigma^{l=1}(p) = -i\frac{e^2}{8} p \quad (12)$$

$$i\Gamma_{\nu}^{l=1}(p, q) = i\frac{e^2}{8} \left[\frac{p_{\nu} + q_{\nu}}{p + q} + \Gamma_{\nu}^{t(l=1)}(p, q) \right] \quad (13)$$

$$\begin{aligned} \Gamma_{\nu}^{t(l=1)}(p, q) = & -2 \frac{p_{\mu}(q^2 - p \cdot q) + q_{\nu}(p^2 - p \cdot k)}{(p + q)|p - q|(p + q + |p - q|)} - \epsilon_{\nu\sigma\eta}(q - p)_{\sigma} \gamma_{\eta} \frac{1}{p + q + |p - q|} \\ & + [(pq + p \cdot q) \epsilon_{\nu\sigma\eta} \left(\frac{q_{\sigma}}{q} - \frac{p_{\sigma}}{p} \right) + \left(\frac{q_{\nu}}{q} + \frac{p_{\nu}}{p} \right) \epsilon_{\sigma\tau\eta} q_{\sigma} p_{\tau}] \gamma_{\eta} \frac{1}{(p + q + |p - q|)^2} \end{aligned} \quad (14)$$

where $P_{\mu\nu} = \delta_{\mu\nu} p^2 - p_{\mu} p_{\nu}$ and $p = |p|$. These in Eq.(12) are well known, while Eqs.(13) and (14) are new results.

Now we consider the general forms of $\Pi_{\mu\nu}(p)$ and $S^{-1}(p)$. By the dimension argument, the gauge symmetry and the (global) conformal symmetry, we find that the inverse two-point functions take the forms

$$\Pi_{\mu\nu}(p) = \Pi_e P_{\mu\nu}/p + \tilde{\Pi}_o \epsilon_{\mu\nu\sigma} p_\sigma \quad (15)$$

$$S^{-1}(p) = Ap \cdot \gamma + Bp \quad (16)$$

where Π_e , $\tilde{\Pi}_o$, A and B are independent, dimensionless, and finite constants. Accordingly, we have

$$\Delta_{\mu\nu}(p) = \frac{1}{\Pi_e(1+\theta^2)p^3} (P_{\mu\nu} - \theta p \tilde{\Pi}_o \epsilon_{\mu\nu\sigma} p_\sigma) \quad (17)$$

$$S(p) = \frac{1}{A^2 + B^2} \frac{Ap \cdot \gamma - Bp}{p^2} \quad (18)$$

with $\theta = \tilde{\Pi}_o/\Pi_e$. Eqs.(17) and (18), involve no more singularity except the pole at $p^2 = 0$, which is consistent with the argument given above that the matter and photon remain massless.

Each of the DS equations (7) and (8) consists of two independent equations due to the two-by-two γ -matrices. Contracting Eq.(7) with $P_{\mu\nu}(p)$ and with $\epsilon_{\mu\nu\tau} p_\tau$, respectively, we obtain

$$2p^3 \Pi_e = -e^2 P_{\mu\nu}(p) \int \frac{d^3 k}{(2\pi)^3} \gamma_\mu S(k+p) \Gamma_\nu(k+p, k) S(k) \quad (19)$$

$$2p^2 (\tilde{\Pi}_o - 1) = -e^2 \epsilon_{\mu\nu\tau} p_\tau \int \frac{d^3 k}{(2\pi)^3} \gamma_\mu S(k+p) \Gamma_\nu(k+p, k) S(k) \quad (20)$$

Similarly, taking the trace of Eq.(8) and the trace of Eq.(8) multiplied by $p \cdot \gamma$, we obtain

$$2pB = -e^2 Tr \int \frac{d^3 k}{(2\pi)^3} \gamma_\mu S(k+p) \Gamma_\nu(k+p, p) \Delta_{\nu\mu}(k) \quad (21)$$

$$2p^2(A-1) = e^2 Tr \int \frac{d^3 k}{(2\pi)^3} p \cdot \gamma \gamma_\mu S(k+p) \Gamma_\nu(k+p, p) \Delta_{\nu\mu}(k) \quad (22)$$

Our problem is to solve the constants A , B , Π_o and Π_e from the set of Eqs.(19-22). For this purpose, we need to know the three-point function, $\Gamma_\nu(p, q)$.

receives the decomposition:

$$\Gamma_\nu(p, q) = \Gamma_\nu^L(p, q) + \Gamma_\nu^T(p, q) \quad (23)$$

The longitudinal part, $\Gamma_\nu^L(p, q)$, is completely determined by the inverse fermion two-point function via the well-known Ward-Takahashi identity for the $U(1)$ symmetry

$$(p - q)_\nu \Gamma_\nu^L(p, q) = S^{-1}(p) - S^{-1}(q) \quad (24)$$

Using Eq.(16), we have

$$\Gamma_\nu^L(p, q) = A\gamma_\nu + B\frac{p_\nu + q_\nu}{p + q} \quad (25)$$

But the transverse part of the vertex function, $\Gamma_\nu^T(p, q)$, satisfying

$$(q - p)_\nu \Gamma_\nu^T(p, q) = 0 \quad (26)$$

is less constrained. The simplest ansatz $\Gamma_\nu^T = 0$ does not work as the pure longitudinal vertex Eq.(25) has been found to introduce singularities into Eqs.(19-22). As a matter of fact, even at one loop level of the perturbation theory, from Eq.(13) we have seen $\Gamma_\nu^T \neq 0$.

In [34] and [35], the form of the tensor structure of the transverse part of the vertex in (3+1) dimensional QED was discussed, and eight linearly independent terms for the transverse vertex were given. The analysis in [34] and [35] applies to the 2+1 dimensional CS gauge theory as well. In particular, one of the eight transverse vectors, $T_8 = -\gamma_\nu p_\eta q_\tau \sigma_{\eta\tau} + p_\nu q \cdot \gamma - q_\nu p \cdot \gamma$ with $\sigma_{\eta\tau} = \frac{1}{2}[\gamma_\eta, \gamma_\tau]$, turns out to be a pure axial vector. In (2+1) dimensions, it can be written $T_8 = W_\nu = \epsilon_{\nu\sigma\eta} p_\sigma q_\eta$. We would require the vertex function to be 1) gauge invariant so that Eq.(26) holds; 2) the charge conjugation invariant such that it satisfies (the charge conjugation operator $C = \gamma_2$):

$$C^{-1}\Gamma_\nu(p, q)C = [\Gamma_\nu(-q, -p)]^t \quad (27)$$

and 3) free of kinematic singularities [34]. Then the ansatz we will make for the transverse vertex is:

$$\begin{aligned} \Gamma_\nu^T(p, q) = & BQ_\nu \frac{2}{(p+q)|p-q|(p+q+|p-q|)} + C\Gamma_{\nu\sigma}^\epsilon(p, q)\gamma_\sigma \\ & + DQ_\nu \frac{(p+q) \cdot \gamma}{pq|p-q|^2} + EQ_\nu \frac{W \cdot \gamma}{pq|p-q|^3} + FW_\nu \frac{1}{|p-q|(p+q+|p-q|)} \end{aligned} \quad (28)$$

where

$$Q_\nu(p, q) = p_\nu q \cdot (p - q) - q_\nu p \cdot (p - q), \quad W_\nu(p, q) = \epsilon_{\nu\eta\tau} p_\eta q_\tau \quad (29)$$

$$\Gamma_{\nu\sigma}^\epsilon(p, q) = [(pq + p \cdot q)\epsilon_{\nu\eta\sigma}(\frac{q_\eta}{q} - \frac{p_\eta}{p}) - (\frac{q_\nu}{q} + \frac{p_\nu}{p})W_\sigma] \frac{1}{pq} \quad (30)$$

Note that the forms of the first two terms in Eq.(28) have been seen in the one loop correction Eq.(14) and the other terms might come from higher loop corrections, though we are seeking a non-perturbative solution. The constants C, D, E and F will be fixed by demanding that the ultraviolet divergence in the set of Eqs.(19-22) cancel out.

With the ansatz for the full vertex function given in Eqs.(23), (25) and (28), the set of equations (19-22) for the variables A , B , Π_e , and $\tilde{\Pi}_o$ are highly non-linear and inhomogeneous. The general solutions of the set of equations will be discussed elsewhere. Here, to obtain the critical statistical charge for the superconduct phase, we use the condition $\tilde{\Pi}_o = 0$ (and then $\theta = 0$).

Substituting Eqs.(17), (18), (25) and (28) in Eqs.(19-22) and performing the integrations over k with the regularization by dimensional reduction, we obtain a set of algebra equations (with $\tilde{\Pi}_o = 0$):

$$\Pi_e = \frac{e^2}{16(A^2 + B^2)^2} [-A^3 - \frac{4}{\pi^2} AB^2 - \frac{8}{\pi^2} A^2 D + B^2 D - (\frac{4}{\pi^2} - 1) A^2 F] \quad (31)$$

$$1 = \frac{e^2}{32(A^2 + B^2)^2} [(2 - \frac{8}{\pi^2}) A^2 B + \frac{16}{\pi^2} A^2 E + B^2 E + \frac{8}{\pi^2} ABF] \quad (32)$$

$$B = \frac{Ae^2}{48\Pi_e(A^2 + B^2)} [(3 - \frac{12}{\pi^2}) B - \frac{12}{\pi^2} C + \frac{8}{\pi^2} E] \quad (33)$$

$$A = 1 + \frac{e^2}{64\Pi_e(A^2 + B^2)} [-\frac{32}{\pi^2} B^2 + \frac{48}{\pi^2} BC - \frac{16}{\pi^2} AD + BE + (\frac{16}{\pi^2} - 4) AF] \quad (34)$$

with the finiteness conditions

$$0 = 5(B^2 - AF) + 2BE \quad (35)$$

$$0 = A^2B - 2(A^2 + B^2)C - 2ABD + ABF \quad (36)$$

$$0 = 2BD - AE \quad (37)$$

$$0 = 2A^2 + B^2 + 2AF \quad (38)$$

Using the conditions (35-38), eliminating Π_e by Eq.(31), and setting $A = xB$ in the Eqs. (32-34), we obtain

$$0 = 520x^4 + (1132 - 102\pi^2)x^2 + 264 - 81\pi^2 \quad (39)$$

$$0 = [80x^6 - (58\pi^2 + 72)x^4 - (37\pi^2 + 288)x^2 + 7\pi^2 + 208]B \\ + 2x(-80x^4 + (26\pi^2 - 152)x^2 + 19\pi^2 + 16) \quad (40)$$

$$0 = [160x^4 + (2\pi^2 + 304)x^2 + 15\pi^2 + 16]e^2 + 128\pi^2(x^2 + 1)^2B \quad (41)$$

The unique physical solution is (in the other solutions, either x^2 or e_c^2 has a wrong sign)

$$e_c = \pm 2.106 \quad (42)$$

companying with $A = 0.5404$, $B = -0.5692$ and $\Pi_e = -0.3773$.

It is interesting to notice that the critical statistical charge obtained here with the non-perturbative method differs from the two-loop result by merely 5.6%.

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References

- [1] S. Deser, R. Jackiw and R. Templeton, Phys. Rev. Lett. 48 (1982) 975; Ann. Phys. (NY) 140 (1982) 372.
- [2] F. Wilczek, Phys. Rev. Lett. 48 (1982) 1144; 49 (1982) 957.

- [3] D. Arovas, J.R. Schriber, F. Wilczek and A. Zee, Nucl. Phys. B251 (1985) 117.
- [4] A.J. Niemi and G.W. Semenoff, Phys. Rev. Lett. 54 (1985) 2166.
- [5] G.W. Semenoff, Phys. Rev. Lett. 61 (1988) 517.
- [6] E. Fradkin, Phys. Rev. Lett. 63 (1989) 322.
- [7] Y. Matsuyama, Phys. Lett. 228B (1989) 99.
- [8] *The Quantum Hall Effect*, ed. by R. Prange and S. Girvin, Springer-Verlag (1987).
- [9] A. Khurana, *Experiments Provide Evidence for the Fractional Charge of Quasiparticles*, Phys. Today, Jan. 1990.
- [10] Y.-S. Wu, *Topological Aspects of the Quantum Hall Effect in Physics, Geometry, and Topology* ed. by H. C. Lee, Plenum Publishing (1990).
- [11] R. B. Laughlin, Science 242 (1988) 525; V. Kalmeyer and R.B. Laughlin, Phys. Rev. Lett. 59 (1987) 2095.
- [12] A.L. Fetter, C.B. Hanna and R.B. Laughlin, Phys. Rev. B39 (1989) 9679.
- [13] Y.H. Chen, F. Wilczek, E. Witten and B.I. Halperin, Int. J. Mod. Phys. B3 (1989) 1001.
- [14] X.G. Wen and A. Zee, Phys. Rev. Lett. 62 (1989) 2873.
- [15] M.P.A. Fisher and D.H. Lee, Phys. Rev. Lett. 63 (1989) 903.
- [16] For a modern review, see for instance, *Fractional Statistics and Anyon Superconductivity*, ed. by F. Wilczek, World Scientific (1990).
- [17] T. Banks and J. Lykken, Nucl. Phys. B336 (1990) 500.

- [18] S. Randjbar-Daemi, A. Salam and J. Strathdee, Nucl. Phys. B340 (1990) 403.
- [19] J.D. Lykken, J. Sonnenschein and N. Weiss, Phys. Rev. D42 (1990) 2161; Int. J. Modn. Phys. A6 (1991) 1335.
- [20] Y. Hosotani, Int. J. Modn. Phys. B7 (1993) 2219.
- [21] E.J. Ferrer and V. de la Incera, Int. J. Modn. Phys. B9 (1995) 3585.
- [22] S. Coleman and B. Hill, Phys. Lett. 159B (1985) 184.
- [23] G.W. Semenoff, P. Sodano and Y-S. Wu, Phys. Rev. Lett. 62 (1989) 715.
- [24] W. Chen, Phys. Lett. 215B (1990) 415; V.P. Spiridonov and F.V. Tkachov, Phys. Lett. 260B (1991) 109.
- [25] W. Chen, M.P.A. Fisher and Y.-S. Wu, Phys. Rev. B48 (1993) 13749.
- [26] T. Appelquist, D. Nash and L.C.R. Wijewardhana, Phys. Rev. Lett. 60 (1988) 2757
- [27] D. Atkinson, P.W. Johnson and P. Maris, Phys. Rev. D43 (1990) 602.
- [28] D. Nash, Phys. Rev. Lett. 62 (1989) 3024.
- [29] N. Dorey and N.E. Mavromatos, Phys. Lett. B266 (1991) 163; Nucl. Phys. B386 (1992) 614.
- [30] W.-H. Ke and J.-K. Kim, Phys. Rev. D50 (1994) 5398.
- [31] W. Chen, G.W. Semenoff and Y.-S. Wu, Mod. Phys. Lett. A5 (1990) 1833; Phys. Rev. D44 (1991) R1625; 46 (1992) 5521.
- [32] R. Pisarski and S. Rao, Phys. Rev. D32 (1985) 2081.

- [33] J. Collins, *Renormalization*, Cambridge University Press (1984).
- [34] J. Ball and T.-W. Chiu, Phys. Rev. D22 (1980) 2542.
- [35] D.C. Curtis and M.R. Pennington, Phys. Rev. D42 (1990) 4165; 46 (1992) 2663; 48 (1993) 4933.